Efficiency of non-renewable energy extraction in Africa

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**Abstract**

This paper evaluates the efficiency of non-renewable energy resource extraction in Africa through the lens of the Hotelling rule. Using both continental-level data and country- specific analysis for the five largest producers of natural gas, oil, and coal, it investigates whether current extraction paths reflect economically optimal depletion behavior. In addition, the study estimates the optimal depletion period using a dynamic optimiza- tion framework that accounts for cost structures and market parameters. Findings reveal substantial divergence from the Hotelling model at the continental level, with estimated firm discount rates near zero, in stark contrast with actual real interest rates in African markets. Across resources, optimal depletion times are significantly longer than current practices suggest. Coal exhibits the longest optimal depletion hori- zon (372 years), followed by Natural gas (351 years) and oil (248 years), contrasting sharply with observed averages of 90, 74, and 47 years, respectively. Nevertheless, some countries—such as Angola, Nigeria, Libya, and Zimbabwe—display extraction paths that more closely align with intertemporal efficiency, while others—including Al- geria, Egypt, Mozambique, and South Africa—exhibit deviations driven by short-term extraction incentives or institutional constraints. These results underscore the urgent need for improved governance, stronger economic incentives, and sustained innovation to bring resource management in Africa closer to its long-run economic optimum. The findings offer critical insights for policymakers seeking to balance revenue mobilization, energy security, and intergenerational equity.

***Keywords—*** Non-renewable energies, Hotelling rule, Scarcity rent, Stock effect, Tech- nological progress, Optimal extraction

***JEL classification codes:*** Q31, Q32, Q35, C61

# Introduction

Africa is abundant in both renewable and non-renewable natural resources. Its substantial energy reserves make it a significant player in the global energy market. The continent holds about 30% of the world’s oil reserves, 13% of its natural gas reserves, and 25% of its mineral reserves.

Energy resources—oil, natural gas, and coal—are central to the economies of many African countries, driving economic growth, government revenues, and export earnings. On average, natural resources contribute approximately 20-30% to Africa’s GDP, with oil, gas, and coal playing a dominant role in resource-rich nations. For example, oil accounts for over 30% of Angola’s GDP and around 10% of Nigeria’s GDP, while coal contributes significantly to South Africa’s economy, representing about 7% of its GDP (World Bank, 2023). These resources also form a substantial share of government revenues, often exceeding 20% and reaching up to 75% in oil-dependent countries like Angola and Nigeria (World Bank, 2023). Additionally, energy exports are a major source of foreign currency, with oil and gas alone contributing over 50% of total export earnings in many African nations. For instance, oil constitutes more than 90% of Nigeria’s exports, while natural gas accounts for a significant portion of Algeria’s and Mozambique’s export revenues (World Bank, 2023). However, the contribution of these resources varies across countries. Oil-rich nations like Libya and Angola rely heavily on hydrocarbons, while others, such as South Africa, depend more on coal. This diversity highlights the importance of efficient resource management to ensure long-term economic sustainability and stability across the continent.

The objective of this study is twofold. First, it evaluates the efficiency of non-renewable

resource extraction practices in Africa, analyzing whether current extraction behaviors align with the economic theory of depleted resources. Second, it determines the optimal depletion of mineral resources under both current practices and the economic efficiency of depleted

resources. The aim is to provide decision-makers in Africa with projections of their strategic resources, enabling them to adjust their policies and practices to ensure long-term economic sustainability.

The economics of depleted resources was articulated in 1930 by Harold Hotelling. He formulated that a rational firm adjusts the extraction of a natural resource so that the resource rent (market price minus extraction cost) increases at the firm’s discount rate, corresponding to its opportunity cost. This arbitrage principle is referred to as the Hotelling rule. We conduct this analysis both at the continental and country levels, focusing on each resource’s five largest producers. The study examines Algeria, Egypt, Libya, Mozambique, and Nigeria for natural gas. In the case of oil, the analysis includes Algeria, Angola, Egypt, Libya, and Nigeria. Finally, we evaluate extraction practices for coal in Botswana, Mozambique, Nigeria, South Africa, and Zimbabwe.

Some empirical studies have applied such a rule with mixed results. Its application remains rather contextual.[[1]](#footnote-1) It has been established in some cases, such as U.S. oil and gas valuation ([Miller and Upton](#_bookmark20) ([1985](#_bookmark20))) and timber markets ([Livernois et al.](#_bookmark19) ([2006](#_bookmark19))). However, limitations have been emphasized, such as sensitivity to technology and stock effects, and risk factors often ([Young and Ryan](#_bookmark25) ([1996](#_bookmark25)); [Slade and Thille](#_bookmark22) ([2009](#_bookmark22))). Technological progress may lead to a temporary rise in resource price by its ability to offset initial increases in stock cost, a behavior quite common in models accounting for stock effects ([Slade](#_bookmark21) ([1982](#_bookmark21)); [Zimmerman](#_bookmark26) ([1977](#_bookmark26))).

Empirical research on the Hotelling rule in the African context is limited. [Tatoutchoup](#_bookmark23) [et al.](#_bookmark23) ([2022](#_bookmark23)) analyze the oil and gas extraction efficiency in Cameroon while overlooking coal. Their study focuses solely on testing the Hotelling model. In contrast, our research extends

beyond this test by determining the optimal depletion time of the energy resources, consid-

ering both current extraction practices and those predicted by the economic theory dictated by the Hotelling model. This analysis is necessary for African countries to address economic challenges strategically or to adjust to the current extraction path, given the contribution of these resources to their economy.

This paper contributes to the literature in energy economics by enabling empirical es- timation of the optimal duration of the extraction of the depleted resource. This is scarce due to the assessment of the cost function and the complexity of solving differential equa- tions. In addition, the analysis of the African extraction context practice offers a guide for policymakers on managing the resources for economic sustainability.

The results show great divergences from Hotelling’s model for most continental-level re- sources due to inefficiencies, resource depletion rates, and technological advancement. The analysis on a country level uncovers important heterogeneity in extraction efficiencies, espe- cially among the five largest producers of each resource. Aligning extraction practices with sustainability principles is crucial to ensure long-term energy security for the continent. The findings of this study call for time-sensitive policies to extend the time of resource depletion and find solutions to sustainability challenges. Resource depletion time shows significant variability as a function of interest rates and suggests using economic incentives to extend availability.

To achieve this, this study presents methods for estimating cost functions and testing this rule in the second section. It also offers in-depth information about the data set used in the analysis. Section 3 continues with the estimation of cost functions, empirical tests of Hotelling’s model, and estimates of the resource depletion time for observed extraction prac- tices at the continental and country levels. Finally, the study summarizes the key conclusions, their implications for the African energy sector, and some proposed recommendations toward more sustainable ways of managing resources.

# Review of the Theoretical Hotelling Model

Let first review the extended version of the Hotelling model. Consider a firm extracting a depletable resource in a competitive market. Let *p*(*t*) represent the market price at time *t*, and *c*(*q*(*t*)*, Q*(*t*)*, t*) denote the extraction cost, which depends on the current production *q*(*t*), the cumulative production *Q*(*t*), and the state of technology *t*. The resource’s initial stock is *S*0, making the remaining stock at time *t* equal to *S*(*t*) = *S*0 *− Q*(*t*). Let *r* represent the discount rate. The firm’s goal is to maximize its intertemporal profit over the extraction period *T* , which is assumed to be endogenous. This can be expressed as:

|  |  |  |
| --- | --- | --- |
|  |  | (1) |
|  |  | (2) |
|  |  | (3) |

Let *H*(*q*(*t*)*, S*(*t*)*, λ*(*t*)*, t*) = *p*(*t*)*q*(*t*) *− C*(*q*(*t*)*, S*0 *− S*(*t*)*, t*) *− λ*(*t*)*q*(*t*) be the current Hamilto- nian value, where *λ*(*t*) is the co-state variable associated with Equation (2) that defines the dynamics of the stock. The necessary conditions for optimization are as follows:[[2]](#footnote-2)

|  |  |  |
| --- | --- | --- |
|  |  | (4) |
|  |  | (5) |
|  |  | (6) |
|  |  | (7) |

Equation (4) is the static efficiency condition. It defines the scarcity rent of the resource, *λ*(*t*) at each period *t*, as the difference between the unit price and the marginal cost. Equation (5) is the dynamic efficiency condition. It suggests that the marginal unit rent grows at a rate lower than the interest rate, *r*, due to the stock effect *CQ*(*t*). In the absence of a stock effect (*CQ*(*t*) = 0), this condition simplifies to the basic Hotelling’s rule. Equations [(6)](#_bookmark2)-[(7)](#_bookmark2) represent the transversality conditions, which will be used later to determine the optimal depletion time, *T* . The Hotelling model is tested by evaluating the joint null hypothesis: (i) *r* equals the firm’s discount rate; and (ii) *βQ* = 1, where *βQ* is the parameter associated with the stock effect in Equation (5) in a regression model. If this hypothesis cannot be rejected, the data is consistent with the Hotelling model. The null hypothesis reduces to (i) for the basic Hotelling rule.

To perform the Hotelling test, we first need to estimate the scarcity rent. This requires estimating the firm’s cost function, *C*(*q*(*t*)*, Q*(*t*)*, t*). The estimation is performed in discrete time. It is specified as:

|  |  |  |
| --- | --- | --- |
|  |  | (8) |

The parameters *β*0, *β*1, *β*2, *α*, *γ* and *γ*0 are nonnegative and 0 *< β*1 *≤* 1. This specification allows for quadratic and linear specifications (*β*2 = 0). We chose this specification because of the analytical tractability of the solution of the system of differential equations involved in the optimization problem, and it later provides a very good fit of the data. Let Γ =

(*β*0*, β*1*, β*2*, α, γ, γ*0) be the vector of parameters and Γˆ

its estimate. The scarcity rent is

estimated as

*λ*ˆ*t* = *pt − C*ˆ(*qt, Qt*) where

*C*ˆ(*qt, Qt*) = *C*(*qt, Qt,* Γˆ). Finally, the regression

equation for the Hotelling is then:

|  |  |  |
| --- | --- | --- |
|  |  | (9) |

with the marginal cost of extraction with respect to cumulative production at time and

*ϵt* the error term *t*.

# Empirical analysis of the efficient extraction

## Data Description

The study focuses on three major energy resources—natural gas, petroleum, and coal—by examining the five largest producers of each to provide a comprehensive view of Africa’s ex- traction dynamics. For natural gas, the analysis includes Algeria, Egypt, Libya, Mozambique, and Nigeria, which collectively accounted for 85.9% of Africa’s total natural gas production in 2023. In terms of petroleum, the study covers Algeria, Angola, Egypt, Libya, and Nigeria, representing 80.6% of Africa’s petroleum production in 2020. Regarding coal, the analy- sis looks at Botswana, Mozambique, Nigeria, South Africa, and Zimbabwe, with these five countries contributing 99.2% of Africa’s coal production in 2022.

Figure 1: Energy Contribution by Resource



This targeted selection captures the dominant actors in the African energy resource extraction landscape. The periods of analysis are chosen based on the availability and reliability of data on each individual resource. The natural gas data span from 1980 to 2023 and thus cover the evolution of production practices of the five largest producers. While oil covers the period 1973 to 2023, coal spans 1970-2022. These time frames provide a sufficient scale to study long-term trends and the sustainability of resource extraction. The data set includes major variables such as quantity extracted, costs, and prices, which were necessary in calculating scarcity rents and cumulative production. These variables are fundamental for assessing the efficiency of resource extraction and estimating the depletion time of these energy reserves. The definitions, units, and sources of these variables are detailed in Table [1](#_bookmark6). All nominal variables are adjusted to real values using the country’s consumer price index (CPI).[[3]](#footnote-3)

Table 1: Description of Variables and Sources

Real prices Annual prices, 1960 to present, real 2010 US dollars

**Variable Definition Unit Source**

Current extraction Production per commodity and country

US$/metric WBCPD Metric EIA and BGS

Cumulative extraction Cumulative production Metric Computed Real extraction cost Cost of extraction of the resource Metric Computed

Energy resource rents (% of GDP) Energy resource rents are the

difference between the value of crude oil production at regional prices and total costs of production.

Energy resource rents Rent of each commodity produced per country

Percentage WDI

Metric Computed

Real GDP Gross Domestic Product Metric WDI

GDP deflator Ratio of GDP in current local currency to GDP in constant local currency. The base year varies by country.

Real interest rate Average interest rate in constant terms

Index WDI

Percentage WDI

Population Total population Metric WDI

WBCPD: World Bank Commodity Price Data. WDI: World Development Indicators. EIA: U.S. Energy Information Administration for natural gas and petroleum extraction. BGS: British Geological Survey for coal extraction.

Figure [2](#_bookmark7) illustrates the price trends of the three for three energy resources: natural gas, oil, and coal. Table [2](#_bookmark8) provides a summary of the preliminary statistics from the sample data.

Figure 2: Evolution of market prices

20 **Natural Gas**

100

**Petroleum**

250

**Coal**

80 200

15

60 150

**Price, ($)**

**Price, ($)**

**Price, ($)**

10

40 100

5

20 50

0

1970 1980 1990 2000 2010 2020

**Year**

0

1970 1980 1990 2000 2010 2020

**Year**

0

1970 1980 1990 2000 2010 2020

**Year**

It follows from Figure [2](#_bookmark7) that all three resources exhibit a U-shaped trend up to the year 2010. Initially, their prices show a decreasing trend up to the early 2000s, followed by an upward trajectory until 2010. This pattern suggests that technological advancements in extraction methods may have driven prices down initially, while the stock effect—where diminishing reserves increase scarcity—likely contributed to the subsequent price rise. After 2010, the prices of all three resources enter a declining phase, which persists until 2020. However, a notable shift occurs starting in 2020, with significant price jumps observed for natural gas and coal. This increase coincides with the global economic recovery following the COVID-19 pandemic and peaks during the Russo-Ukrainian crisis, which disrupted energy markets.

Table 2: Descriptive Statistics of African Natural Gas, Petroleum, and Coal

|  |
| --- |
| **Variable Natural Gas Petroleum Coal** |
|  | **Unit** | **Mean** | **Std. Dev** | **Unit** | **Mean** | **Std. Dev** | **Unit** | **Mean** | **Std. Dev** |
| Market price | $/MMbtu | 5.29 | 3.07 | $/bbl | 45.79 | 24.52 | $/mt | 61.57 | 34.77 |
| Current extraction | tcf | 5.36 | 2.88 | bbbl | 2.72 | 0.63 | bt | 0.20 | 0.07 |
| Cumulative extraction | tcf | 85.17 | 72.43 | bbbl | 63.91 | 42.17 | bt | 4.29 | 3.22 |
| World demand | tcf | 102.58 | 30.92 | bbbl | 27.39 | 5.84 | bt | 5.35 | 1.80 |
| Total cost | $1b | 58.40 | 43.86 | $1t | 2.25 | 3.88 | $1b | 78.36 | 86.69 |

*Notes:* MMbtu = Million British Thermal Units; tcf = Trillion Cubic Feet; $1b = One Billion Dollars; bbbl = Billion Barrels; bt = Billion Tonnes.

## Efficient extraction analysis

For each energy resource, the Hotelling rule was tested at both the continental level (Africa as a whole) and at the individual country level. The empirical test of the Hotelling rule was conducted separately for each country, rather than using a panel approach. This decision was driven by the recognition that extraction costs can vary significantly between countries due to differences in geological, technological, and economic conditions. By analyzing each country independently, the study ensures a more accurate and context-specific assessment of extraction efficiency and adherence to the Hotelling rule.

### Efficient extraction analysis at Africa-Wide

Table 3: Estimation Results for Natural Gas, Petroleum, and Coal

**Parameters**

**Natural Gas**

Interaction cost function

**Petroleum**

Non linear without stock effect

**Coal**

Non linear without stock effect

*β*ˆ0 203.1\*\*\* 7074.6\*\*\* 4222.8\*\*\* (25.50) (702.75) (416.47)

*β*ˆ1 0.816\*\*\* 0.891\*\*\* 0.902\*\*\*

(0.01) (0.01) (0.01)

*α*ˆ 2.26e-11\*\*\*

(5.35e-12)

R-squared 0.87 0.81 0.85

Observations 37 50 53

**Hotelling model test for Africa**

*r*ˆ -5.62e-4 -1.95e-4 -1.11e-3

(0.01) (2.799e-4) (9.162e-4)

*β*ˆ*Q* -0.584

(9.64)

DW-statistic 2.00 2.00 1.21

R-squared - - -

Observations 35 50 52

Standard errors in parentheses. Significance Codes,’\*\*\*’ 1%, ’\*\*’ 5%, ’\*’ 10%.

Table [3](#_bookmark9) summarizes the estimation results for the cost functions and the Hotelling test for the three energy resources: natural gas, oil, and coal. The analysis proceeded in two main steps: the estimation of cost functions, and the implementation of the Hotelling test based on these estimates.

The initial step involved estimating the cost functions necessary for conducting the Hotelling test. Various specifications of the cost functions, as detailed in Equation (8), were evaluated. The chosen specifications were those whose parameters conformed to eco- nomic theory. For all three energy resources, a non-linear specification was selected. Only for natural gas was the interaction term *α*ˆ is statistically significant, indicating a potential stock effect. The selected cost functions provided an excellent fit for the data, with output

and cumulative output explaining over 82% of the variability in total costs. Additionally, the results revealed significant technological progress, with growth rates of 18.4%, 10.9%, and 9.4% for natural gas, oil, and coal, respectively. These findings underline the role of technological advancements in reducing extraction costs over time.

The estimation of the cost functions enabled the calculation of scarcity rents, which were then used to perform the Hotelling test. This test evaluates the null hypothesis that the market interest rate equals the firm’s discount rate (), representing the opportunity cost of resource extraction. The opportunity cost reflects the potential returns from alternative investments, influencing the rate at which resources are extracted. The results, summarized in Table 3, show that for all three commodities, the estimated parameters—including the

firm’s discount rate () and the stock effect (*β*ˆ*Q*)—are statistically insignificant. This implies

that the estimated discount rates are effectively zero for all energy resources.

To further assess the validity of the Hotelling model, we determined the maximum dis- count rate for each resource at which the null hypothesis could not be rejected at the 5% significance level. The results indicate a maximum discount rate of 0.1% for each of the three resources. These values are significantly lower than the real interest rates observed in African markets, which averaged 4.5% (1980–2023), 3.1% (1973–2023), and 3.1% (1971–2022) for nat- ural gas, oil, and coal, respectively. This discrepancy suggests that the Hotelling model is inconsistent with the data-generating process for these resources. Thus, the findings indicate that the Hotelling model does not align with the extraction dynamics of natural gas, oil, and coal in Africa.

### Efficient extraction analysis Country-Level

The previous analysis has offered a continental overview of extraction trends, but a deeper understanding necessitates examining specific cases at the country level. By focusing on

the five largest producers of each energy resource—representing 90%, 80%, and 100% of African production for natural gas, oil, and coal respectively, we can determine if there are differences in management across countries. This approach will also highlight similarities and divergences in extraction patterns.

Estimated costs for natural gas, as shown in Table [4](#_bookmark10), help identify trends in extraction cost evolution in major producing countries. Only Algeria has a significant stock effect that influences the natural gas extraction process. This is evidenced by the interaction parame-

ters.

Table 4: Estimation Results for Natural Gas

**Parameters**

**Algeria Egypt Libya Mozambique Nigeria**

Interaction cost Quadratic cost Quadratic cost Quadratic cost Quadratic cost without stock effect without stock effect without stock effect without stock effect without stock effect

*β*ˆ0 122.5\*\*\* 55.64\*\* 11.37\*\*\* 48.24 1785.2\*\*\* (11.60) (17.87) (2.77) (24.48) (112.28)

*β*ˆ1 0.838\*\*\* 0.934\*\*\* 0.966\*\*\* 0.851\*\*\* 0.800\*\*\*

(0.01) (0.02) (0.02) (0.04) (0.01)

*β*ˆ2 7.34e-11 2.48e-09 8.63e-09\* 2.09e-09\*\* (7.34e-11) (2.48e-09) (4.09e-09) (7.32e-10)

*α*ˆ0 6.16e-11\*\*\*

(8.43e-12)

R-squared 0.92 0.83 0.79 0.86 0.95

Observations 44 44 44 26 44

**Hotelling model test**

*r*ˆ -6.26e-3 1.74e-3 0.419 5.41e-3 -3.71e-5 (0.02) (0.01) (0.64) (0.03) (3.256e-4)

*β*ˆ*Q* -3.776

(9.74)

DW-statistic 2.00 1.96 2.05 1.91 2.01

R-squared - - - - - Observations 43 43 43 25 43

Max discount rate (%) 0.02 0.02 1.00 0.06 0.22

Average interest rate (%) 1.50 3.23 0.45 12.86 0.39

Standard errors in parentheses. Significance Codes,’\*\*\*’ 1%, ’\*\*’ 5%, ’\*’ 10%.

Technological progress rates vary significantly between countries, with notable advancements

in Nigeria (20%), Algeria (16.2%), and Mozambique (14.9%), illustrating the rapid modern- ization of gas infrastructure. In contrast, Egypt (6.6%) and Libya (3.4%) show lower progress rates, indicating a slower adoption of innovations in the gas sector.

Now let’s turn to the Hotelling test. For the top five natural gas producers in Africa, we cannot reject the null hypothesis that the discount rate is positive. Consequently, we determined the maximum discount rate for each country, at which this hypothesis cannot be rejected at the 5% significance level. These values are recorded in Table [4](#_bookmark10). It turns out that only Libya has a maximum discount rate that aligns the firm’s discount rate with the market real interest rate in the country, resulting in efficient extraction. However, in the other natural gas producing countries (Algeria, Egypt, Mozambique, and Nigeria), the Hotelling test fails.

Regarding oil, the results of the cost function estimations are presented in Table [5](#_bookmark11). Al- though the estimated cost functions for Algeria and Libya incorporate the stock effect, these effects are statistically insignificant.

Moreover, technological progress is particularly pronounced in Egypt (11.9%) and Nigeria (10%), reflecting sustained efforts in innovation and improved extraction techniques. By comparison, Algeria (7.5%), Libya (5.3%), and Angola (3.7%) show more moderate progress rates.

Table 5: Estimation Results for Petroleum

**Parameters**

**Algeria**

**Angola**

**Egypt**

**Libya**

**Nigeria**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | Non linear with stock effect | Non linear without stock effect | Non linear without stock effect | Interaction cost function | Non linear without stock effect |
| *β*ˆ0 | 931.0\*\*\* | 299.9\*\*\* | 2479.9\*\*\* | 155.7\*\*\* | 18432.1\*\*\* |
| *β*ˆ1 | (99.07)0.925\*\*\* | (68.90)0.963\*\*\* | (316.06)0.881\*\*\* | (20.85)0.947\*\*\* | (1634.58)0.900\*\*\* |
|  | (0.01) | (0.01) | (0.01) | (0.02) | (0.01) |
| *α*ˆ |  |  |  | 1.61e-09 |  |
|  |  |  |  | (9.70e-10) |  |
| *γ*ˆ0 | 0.109 |  |  |  |  |
|  | (0.93) |  |  |  |  |
| R-squared | 0.82 | 0.80 | 0.84 | 0.80 | 0.84 |
| Observations | 51 | 51 | 51 | 51 | 51 |
| **Hotelling model test** |  |  |  |  |  |
| *r*ˆ | -0.0894 | -3.52e-3 | -5.59e-4 | -8.31e-3 | -7.43e-5 |
|  | (0.05) | (0.01) | (8.23e-4) | (0.06) | (1.057e-4) |
| *β*ˆ*Q* | 108.5\*\*\* |  |  | -0.998 |  |
|  | (11.88) |  |  | (6.92) |  |
| DW-statistic | 2.36 | 2.03 | 2.00 | 2.07 | 2.00 |
| R-squared | 0.64 | - | - | 4.00e-4 | - |
| Observations | 50 | 50 | 50 | 50 | 50 |
| Max discount rate (%) | 0.00 | 0.01 | 0.00 | 0.11 | 0.00 |
| Average interest rate (%) | 1.50 | -8.65 | 2.64 | -0.33 | -0.81 |

Standard errors in parentheses. Significance Codes,’\*\*\*’ 1%, ’\*\*’ 5%, ’\*’ 10%.

Similarly to the natural gas analysis, we calculated the maximum discount rate for each country as shown in Table [5](#_bookmark11). The results suggest that the extraction paths of oil in Angola, Libya, and Nigeria align with the Hotelling rule, as we cannot reject the null hypothesis that the firm’s discount rate equals the average market real interest rate. However, these findings must be interpreted cautiously. The average real interest rates between 1973 and 2023 were negative in these countries due to prolonged periods of high inflation. For instance, Angola experienced hyperinflation, with rates exceeding 4,000% in 1996, while Nigeria saw inflation peak at 72.8% in 1995. Libya also faced inflation spikes, such as 15.9% in 2013, during

periods of political instability. Periods of high inflation effectively erode the value of money, making it difficult to accurately assess opportunity costs and resource extraction efficiency. The process that generated the oil data contradicts the Hotelling hypothesis in the other oil-producing countries, namely Algeria and Egypt.

Regarding coal, the cost function estimates, detailed in Table [6](#_bookmark12), indicate that the stock effect is not significant for South Africa. Conversely, for Mozambique and Zimbabwe, the stock effect is significant, indicating that cumulative production influences extraction costs, possibly reflecting the declining quality of exploited deposits.

Furthermore, the results reveal significant disparities in technological innovation and cost evolution among producing countries. Botswana (13.1%) and South Africa (10.3%) show high rates of technological progress, suggesting continuous improvements in extraction processes and infrastructure modernization. In contrast, Nigeria (8.5%), Mozambique (1.3%), and Zimbabwe (1.6%) demonstrate more limited progress.

Table 6: Estimation Results for Coal

**Parameters**

**Botswana**

Quadratic cost without stock effect

**Mozambique**

Interaction function

**Nigeria South Africa Zimbabwe**

Non linear Non linear Non linear without stock effect with stock effect with stock effect

*β*0 3048.4\*\*\* 47.29 43616.9\*\*\* 2445.5\*\*\* 47.64\*\*\* (222.30) (1989.10) (213.01) (213.01) (8.12)

*β*1 0.869\*\*\* 0.987\*\*\* 0.915\*\*\* 0.898\*\*\* 0.984\*\*\* (0.01) (0.04) (0.01) (0.01) (0.01)

*β*2 3*.*09e*−*5\*\*\*

(0.00)

*α* 5*.*76e*−*6\*\*\*

(0.00)

*γ* 1.034 0.489\*

(0.64) (0.21)

R-squared 0.96 0.91 0.98 0.90 0.89

Observations 50 48 53 53 53

**Hotelling model test**

*δ −*1*.*68e*−*3 -0.260 *−*1*.*04e*−*4 -0.0353 0.155

(0.00) (0.31) (0.00) (0.07) (0.37)

*βQ* 7.949\*\* 7.411\*\*\* 9.086\*\*\*

(2.92) (1.49) (0.91)

DW-statistic 1.21 1.79 1.22 1.37 2.02

R-squared 0.00 0.14 0.00 0.33 0.67

Observations 49 46 52 52 52

Max discount rate (%) 0.00 0.35 0.00 0.90 0.90

Average interest rate (%) 3.10 12.87 -1.17 3.21 -22.27

Standard errors in parentheses. Significance Codes: ’\*\*\*’ 1%, ’\*\*’ 5%, ’\*’ 10%.

The Hotelling test applied to major coal producers was used to calculate the maximum dis- count rate for each country, as recorded in Table [6](#_bookmark12). The results indicate that coal extraction practices in Nigeria and Zimbabwe align with the Hotelling rule. As with oil, the results for Nigeria and Zimbabwe regarding coal should be interpreted with caution due to periods of high inflation during the considered timeframe. For example, Zimbabwe experienced hy- perinflation in the 2000s, reaching 7,000% in 2007. In Botswana, Mozambique, and South Africa, the Hotelling tests fail, indicating inefficiencies in extraction.

## Resources depletion time

Here, we analyze the duration of energy resource availability under two scenarios: current extraction rates and optimal extraction as predicted by the Hotelling rule of economic theory. This dual analysis provides policymakers with insights into the potential exhaustion dates of resources, enabling them to prepare for a transition and strategically plan future extractions for sustainability.

The depletion time under current extraction refers to the point at which the resource is completely exhausted, denoted as *T* , where the cumulative output equals the total reserve, i.e., *Q*(*T* ) = *S*0. To estimate this, we modeled the cumulative output *Q*(*t*) as a function of time *t*. We found that a quadratic trend, represented as: *Q*(*t*) = *at*2 + *bt* + *c* provides an excellent fit for the data. The estimated values of *a*, *b*, and *c* for each commodity at both the continental and country levels are presented in the appendix.

Under the Hotelling model, depletion time is determined by solving the system of equa- tions derived from the optimal extraction conditions, specifically Equations (4)–(7). Using the cost function from Equation (8) and its partial derivatives, and substituting these into the system, we obtain the following differential equation:[[4]](#footnote-4)

|  |  |  |
| --- | --- | --- |
|  |  | (10) |

subject to: *Q*(0) = 0*, Q*˙ (*T* ) = 0*, Q*(*T* ) = *S*0

where *Q*˙ = *q*. The first two boundarie conditions are used to solve for the integration

constants, while the third condition is crucial for determining the terminal time *T* , i.e., the time of depletion under the Hotelling rule. To solve the differential equation, it is necessary to

establish the relationship between market price and quantity extracted . We assume a linear demand relationship as follows where: , are parameters and is a vector of variables influencing demand (e.g., income levels, prices of substitutes), is a vector of corresponding parameters. The parameters , and are estimated using empirical data. Detailed estimations and the procedure for solving the differential equation are provided in the appendix.

### Africa-Wide depletion time

Figure [3](#_bookmark14) illustrates the depletion times for natural gas, oil, and coal under current extraction practices (solid red line) and the optimal extraction path predicted by the Hotelling model (dashed blue line). The depletion times for natural gas, oil, and coal under current extraction practices are significantly shorter compared to the optimal depletion times predicted by the Hotelling model, which accounts for resource scarcity and economic efficiency. Specifically, under a 5% discount rate, natural gas is projected to be depleted in 74 years under current extraction, whereas the optimal extraction strategy extends the depletion time to 351 years. Similarly, oil is expected to run out in 47 years under current practices but would last 248 years with optimal extraction. For coal, the depletion time under current practices is 90 years compared to 372 years under the optimal strategy. These differences highlight the suboptimal nature of current extraction practices, which prioritize short-term gains over long-term resource availability.

Figure 3: Remaining stock over time

7

105

**Natural Gas**

**Optimal stock Current stock**

6

14

12

104

**Petroleum**

**Optimal stock Current stock**

3

2.5

104

**Coal**

**Optimal stock Current stock**

5 10

**Remaining stock, S(t)**

**Remaining stock, S(t)**

**Remaining stock, S(t)**

2

4 8

1.5

3 6

1

2 4

1 2 0.5

0

0 50 100 150 200 250 300 350

**Time, t (year)**

0

0 50 100 150 200 250

**Time, t (year)**

0

0 50 100 150 200 250 300 350

**Time, t (year)**

Additionally, as shown in Figure [4](#_bookmark15), the optimal depletion time is sensitive to changes in the interest rate, decreasing as the interest rate increases, which aligns with the Hotelling model’s predictions. For example, for an interest rate change from 4% to 16%, the optimal depletion times decrease from 380 to 320 years for natural gas, 280 to 220 years for oil, and 400 to 340 years for coal. Thus, as the interest rate increases, the gap between current practices and the optimal extraction path narrows. This occurs because higher interest rates reduce the present value of future resource availability, making it economically rational to extract resources more quickly under the Hotelling model.

Figure 4: Terminal time vs interest rate

400

**Current period Optimal period**

350

300

**Terminal time (Year)**

250

200

150

100

**Gas**

300

**Current period Optimal period**

250

200

**Terminal time (Year)**

150

100

50

**Petroleum**

450

**Current period Optimal period**

400

350

**Terminal time (Year)**

300

250

200

150

100

**Coal**

50

0.02 0.04 0.06 0.08 0.1 0.12 0.14 0.16

**Interest rate (r)**

0

0.02 0.04 0.06 0.08 0.1 0.12 0.14 0.16

**Interest rate (r)**

50

0.02 0.04 0.06 0.08 0.1 0.12 0.14 0.16

**Interest rate (r)**

### Country-Level depletion time

Building on the methodology used in the Africa-wide analysis, we performed a similar eval- uation of terminal times under both current time and optimal scenarios. The analysis of natural gas, petroleum, and coal terminal times for the top five countries is summarized in Table [7](#_bookmark16), which provides both current time and optimal terminal times at interest rates from 4% to 10%.

Table 7: Depletion time for natural gas, petroleum, and coal

|  |
| --- |
| **Resource Country Current time Optimal depletion time Stock** |
|  |  | (years) | **4%** | **5%** | **6%** | **7%** | **8%** | **9%** | **10%** |  |
|  | Algeria | 55 | 103 | 94 | 86 | 82 | 81 | 74 | 63 | 163 402 |
|  | Egypt | 48 | 96 | 81 | 74 | 70 | 67 | 64 | 61 | 64 722 |
| **Natural gas** | Libya | 84 | 98 | 84 | 77 | 73 | 70 | 68 | 67 | 54 600 |
|  | Mozambique | 132 | 200 | 184 | 177 | 172 | 169 | 167 | 154 | 102 743 |
|  | Nigeria | 95 | 219 | 204 | 196 | 192 | 189 | 185 | 182 | 209 041 |
|  | Algeria | 28 | 130 | 114 | 107 | 102 | 99 | 95 | 92 | 12 202 |
|  | Angola | 36 | 75 | 44 | 41 | 39 | 38 | 36 | 35 | 7 780 |
| **Petroleum** | Egypt | 26 | 44 | 42 | 40 | 39 | 38 | 36 | 34 | 3 300 |
|  | Libya | 110 | 120 | 106 | 99 | 95 | 92 | 89 | 86 | 48 369 |
|  | Nigeria | 49 | 42 | 37 | 34 | 31 | 29 | 28 | 27 | 36 896 |
|  | Botswana | 325 | 457 | 457 | 445 | 442 | 440 | 438 | 438 | 1 830 |
|  | Mozambique | 192 | 225 | 233 | 225 | 220 | 219 | 217 | 215 | 1 975 |
| **Coal** | Nigeria | 43 | 45 | 52 | 48 | 47 | 45 | 43 | 42 | 2 363 |
|  | South Africa | 55 | 525 | 511 | 502 | 497 | 492 | 489 | 487 | 10 905 |
|  | Zimbabwe | 174 | 201 | 179 | 167 | 160 | 155 | 151 | 148 | 553 |

*Note:* Natural gas and petroleum stock for year 2021 in tcf and bbbl respectively. Coal stock for year 2022 in bt.

The results reveal significant disparities in how different African countries’ extraction rates align with economically optimal depletion times. As observed, the terminal depletion time systematically decreases as the discount rate increases, a trend consistent with findings at

the continental level, where higher discount rates lower the economic value of future reserves, incentivizing faster extraction. Such outcomes are particularly relevant for countries where capital costs are high or where extraction decisions are influenced by short-term economic considerations.

Furthermore, extraction practices adhering to the Hotelling rule generally lead to longer terminal depletion times compared to those that do not. Countries that apply this princi- ple are likely to sustain resource availability over the long term while ensuring consistent economic returns. Among natural gas producers, Libya stands out with a current depletion time of 84 years that aligns well with its optimal depletion times across different discount rates. In the petroleum sector, Libya, Angola, and Nigeria demonstrate economically optimal extraction practices, with current depletion times that align well with their optimal ranges across various discount rates. In the coal sector, Nigeria and Zimbabwe exhibit extraction strategies aligned with economic efficiency, with current depletion times that align well with their optimal ranges across various discount rates. Countries demonstrating economically optimal extraction practices typically follow robust resource management frameworks that effectively balance short-term economic gains with long-term sustainability. These practices may reflect a strategic approach that integrates economic rationality with adaptive extraction methods.

In contrast, extraction practices that do not align with the Hotelling rule generally result

in slower-than-optimal extraction, reflecting conservative approaches or capacity limitations. For natural gas, Algeria, Egypt, Mozambique, and Nigeria all display slower-than-optimal extraction practices, with current depletion times that are systematically lower than their optimal ranges. In the petroleum sector, Algeria and Egypt also show slower-than-optimal extraction, with current depletion times consistently below optimal values. In the coal sec- tor, Botswana, Mozambique, and South Africa also exhibit slower-than-optimal extraction

practices, indicating an overly cautious approach that may reduce potential economic ben- efits. Countries exhibiting slower-than-optimal extraction practices may adopt conservative management strategies aimed at preserving reserves or may face financial and institutional constraints that limit extraction capacity. While such cautious approaches can support sus- tainability, they may reduce economic gains if market conditions change or demand decreases.

# Discussion

This study highlights considerable variation in how African countries manage the extraction of non-renewable energy resources. While economic models set out clear predictions about what an optimal depletion path should look like, the realities on the ground tell a different story. Actual extraction strategies often veer off course, and understanding why means looking beyond general trends to the details—how each country, and each resource, plays out in practice.

Petroleum offers a good starting point. A few countries—Angola, Libya, and Nige- ria—seem to follow extraction paths broadly in line with the Hotelling rule. The earlier tests confirmed this, but the more pressing question is: what’s different about these cases? Why do these countries, despite facing governance challenges, show patterns closer to theo- retical expectations? One explanation lies in how their oil sectors are organized. In Nigeria and Angola, for instance, production-sharing agreements and joint ventures give the state a stronger hand in planning and decision-making. That structure may not solve every problem, but it does allow for longer-term thinking. Libya, for its part, has a long history of state control over oil infrastructure, which might explain its relatively measured depletion even amid political turmoil.

The contrast with Egypt and Algeria is stark. In both countries, oil is being extracted

much faster than what would be considered efficient. Egypt’s depletion timeline, in par- ticular, is almost half of the optimal estimate. It’s hard not to read this as a response to immediate fiscal needs—pressures that often crowd out long-term strategy. Budget short- falls, weak institutional capacity, and a lack of investment in upstream infrastructure are all likely factors. And when global oil prices swing wildly, governments sometimes feel forced to extract as much as possible, while prices are favorable. This isn’t just speculation—economic literature has consistently shown that short-term revenue needs often override intertemporal efficiency in resource-dependent economies, particularly when political budgetary pressures are high ([van der Ploeg and Venables](#_bookmark24) ([2011b](#_bookmark24))).

Natural gas tells a slightly different story, though similar pressures are still at play. Libya again stands out for having an extraction pattern that comes close to the efficient path. The explanation here may be simple: its gas sector hasn’t been developed as aggressively as its oil sector, which leaves more room for a conservative approach. On the other end, countries like Nigeria, Egypt, and Mozambique diverge sharply. Nigeria and Egypt both extract gas at a rapid pace—mirroring what we see in oil—which suggests that short-run fiscal needs are shaping policy across the board. Mozambique is unusual for the opposite reason: its gas sector is moving too slowly. While that might sound like caution, it likely reflects bottlenecks—underinvestment, regulatory delays, or hesitancy tied to uncertain long- term LNG prices.

Coal-producing countries show even more variation. In Zimbabwe and Nigeria, extraction aligns more closely with the Hotelling rule. That said, Zimbabwe’s data comes with a wide margin of uncertainty, so interpretations should be cautious. Nigeria’s alignment could simply reflect scale: its coal sector is small, and demand is modest, naturally keeping extraction in check. South Africa and Botswana fall at the opposite end, extracting coal at a slower-than- optimal rate. In South Africa’s case, this might be a legacy of energy policy centered on

self-sufficiency and reserve preservation, particularly under state-owned enterprises. Those policies may have once served a strategic purpose, but now they could be limiting potential economic gains. Botswana’s slower pace is harder to pin down. For a country that depends heavily on coal for domestic energy, this pattern may point to technical limitations or perhaps an overly cautious regulatory framework.

Looking at the broader picture, it becomes clear that suboptimal extraction isn’t just a matter of how much oil, gas, or coal a country has. It reflects deeper issues: institutional ar- rangements, political constraints, and macroeconomic vulnerabilities. In many cases, govern- ments are simply reacting to pressing short-term needs—debt repayments, budget pressures, or foreign exchange shortages. Price volatility only adds to this, pushing states to extract more while prices are high, rather than wait. Real Options Theory captures this logic well: under uncertainty, actors tend to act quickly to lock in value, even if it means straying from long-term efficiency.

One issue that rarely gets enough attention is information asymmetry. In several coun- tries, governments have limited capacity to monitor what companies are doing on the ground. If contracts are vague or opaque, and enforcement is weak, firms may extract faster than agreed, knowing there’s little chance of being caught. This undermines any attempt at coor- dinated or rational depletion strategies.

Taken together, these patterns suggest a need for reforms that are both ambitious and grounded in each country’s realities. There’s no single blueprint. That said, a few priori- ties stand out: improving oversight, bringing transparency to contracts, and building fiscal tools—like stabilization funds—that give governments breathing room when prices fall. And any serious long-term strategy will have to reckon with environmental risks and intergenera- tional fairness. Extraction policies shouldn’t just meet today’s needs; they need to safeguard tomorrow’s opportunities as well. Getting there will require stronger institutions, better

planning, and a shift in mindset—from short-term survival to long-term stewardship.

# Conclusion

This study has evaluated the efficiency of non-renewable energy extraction practices in Africa measured against the Hotelling rule. The methodological framework employed for the analysis loads upon cost function estimation and optimal depletion modelling to measure current extraction behaviours for economic sustainability.

The study’s findings reveal prevalent inefficiencies. Current extraction practices are illus- trated to deviate continentally from a trajectory of optimum extraction, which is predictable from the theoretical framework. Although some countries, such as Libya for natural gas and oil, Angola and Nigeria for oil, Zimbabwe and Nigeria for coal, appear to extract according to the Hotelling rule, there are more countries that appear either to be extracting too quickly or too slow, relative to the economically efficient trajectory. As suggested, these trends may be a product of a priority for short-term revenues, either as a product of fiscal necessities, or as a product of political constraints, or sometime as a result of inadequate institutional capacity.

The study’s findings reveal a striking gap between current and optimal depletion time. Under current extraction practices, gas is projected to be depleted in 74 years, oil depleted in 47 years, and coal in 90 years, all continentally. However, under optimal extraction practices, gas will not be depleted for 351 years, oil will not be depleted for 248 years, and coal will not be depleted for 372 years respectively. The discrepancy speaks to the inefficiency of extraction practices overall and, exclusively where oil is a key export, extraction practices appear to be accelerated to facilitate immediate economic needs.Another significant conclusion relates to the sensitivity of optimal depletion time to the discount rate, whereby an increase in

interest rate leads to a shorter optimal horizon which narrows the gap to current practices. This indicates the importance of macroeconomic variables (e.g., inflation, nominal interest rate volatility) on long-run extraction approaches. For instance, countries characterized by high inflation or having historically negative real interest rates, Angola, Libya or Nigeria, may appear consistent with the Hotelling rule as a result of the distorted nature of inflation rather than well-thought out, long-term planning.

In terms of methodology, this paper proposes an applied framework for assessing extrac- tion efficiency and optimal depletion trajectories specifically for Africa. It constructs an empirical foundation through estimating a structural cost characterization, incorporates a dynamic model, and applies a bootstrap-based approach to estimate robust estimates and confidence intervals. This framework has the potential to be followed by policymakers or an- alysts in other regions/resource sectors to assess extraction paths and inform sustainability considerations.

The findings have various policy implications. First, the evidence of systemic deviation from intertemporal efficiency suggests the need for stronger governance of extractive indus- tries. Resource rich countries need to nurture organizations that support forward-thinking planning, fiscal stabilization and reinvestment of rents into productive assets. Second, ex- traction choices should be structured, not solely on current needs of satisfying consumption however to ensure intergenerational equity is honored and preserved. Third, incorporating discounting, stock effects, and technological change into our modelling framework provides an avenue for coordinating strategic reform of extraction regimes to support long-term economic goals.

Lastly, follow-up research should develop the modelling framework to explicitly include environmental and social externalities of resource extraction, uncertainty related to resource prices and production for exploration dynamics. Empirical research should further examine

the role of multinational firms, contractual structures, and geopolitical dependencies in shap- ing extraction trajectories for Africa. Appreciating these elements is critical to understanding the constraints and opportunities for sustainable resource management in the continent.

Overall, this research demonstrates the desire for change in extraction policies for Africa. Achieving both efficient and sustainable use of non-renewable resources will necessitate more than basing revisions on economic considerations, but rather a combination of significant strategic policy reform that captures the long-term development vision for the continent.

# APPENDIX

### Appendix A: Estimate of the demand function

The demand for non-renewable energy resources—such as natural gas, oil, and coal—is in- fluenced by a range of economic factors. According to classical economic theory, the market for these resources is characterized by a simultaneous determination of supply and demand, meaning that both price and quantity are resolved endogenously within the market.

To accurately capture these interactions, we adopt a system of simultaneous equations representing the inverse demand and inverse supply functions. This framework allows us to model the complex, interdependent relationship between supply and demand as follows:

Here, is the market price of the resource, and is the equilibrium quantity exchanged at time *t*. The vector denotes demand-side variables such as prices of substitute energy sources (e.g., oil and coal in the case of gas demand), world GDP as a proxy for income levels, and domestic consumption per capita to account for structural consumption patterns. These variables capture the economic environment and behavioral responses that influence energy demand across different national contexts..

On the supply side, we consider the real interest rate *r*(*t*)[[5]](#footnote-5), which plays a crucial role in determining the cost of capital and thus affects the profitability of energy production over time. The parameter captures the downward-sloping nature of the demand curve, while reflects the typical upward-sloping nature of the supply curve. The coefficient

 is expected to be positive under the assumption that higher real interest rates increase

the cost of capital and production. The vector contains the coefficients associated with the explanatory variables in the demand function.

To account for the evolving nature of energy demand over time—driven by economic growth, industrialization, and demographic trends—we model the time dynamics of the vari- ables in using an exponential specification:

This formulation reflects the fact that economic drivers of demand do not remain static but tend to grow (or decline) over time, often at a constant proportional rate. This assumption is particularly relevant given the long time horizon involved in modeling terminal resource depletion. The exponential form is thus consistent with both theoretical growth models and observed long-term energy demand patterns.

Because price and quantity are jointly determined, using standard estimation techniques such as Ordinary Least Squares (OLS) would lead to biased and inconsistent estimates due to the endogeneity of price. To correct for this issue, we employ an instrumental variable (IV) approach, which allows us to isolate exogenous variations in price and obtain more reliable estimates.

### Appendix B: Depletion time under optimal extraction

The optimal depletion time of a natural resource is determined by solving a second-order nonhomogeneous differential equation derived from the system governing the resource ex- traction path. Starting from the dynamic form of Hotelling’s rule and incorporating the cost function , we obtain:

|  |  |  |
| --- | --- | --- |
|  |  | (11) |

We adopt the cost function:

|  |  |  |
| --- | --- | --- |
|  |  |  |

Substituting derivatives and using , we obtain:

|  |  |  |
| --- | --- | --- |
|  |  | (12) |

Given the inverse demand function:

,

Substituting into the (12):

|  |  |
| --- | --- |
|  | (13) |

Letting *y* = *Q*, we obtain:

|  |  |  |
| --- | --- | --- |
|  |  | (14) |

with:

Boundary conditions:

To solve this nonhomogeneous second-order differential equation, we decompose the general solution into two parts: the general solution of the associated homogeneous equation and a particular solution to the nonhomogeneous equation. That is,

where solves the homogeneous equation, and is any particular solution that satisfies the full equation.

In what follows, we consider two cases separately:

**CASE 1**:

**Homogeneous solution:**

**Particular solution:**

**Coefficients:**

**CASE 1**:

**Homogeneous solution:**

**Particular solution:**

**Coefficients:**

Table 8: Summary Table of Each Case

|  |  |  |
| --- | --- | --- |
| **Component** |  |  |
| Homogeneous solution |  |  |
| Particular solution form |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

## Determination of Terminal Time

The optimal depletion time satisfies:

### Appendix C: Estimation of cumulative and demand functions

Table 9: Cumulative Extraction Function Estimate

|  |  |  |  |
| --- | --- | --- | --- |
| **Variable** | **Gas** | **Oil** | **Coal** |
| *a*ˆ0 | 2895566.1\*\*\* | 658077.9 | -251019.3\*\*\* |
|  | (800254.80) | (729529.33) | (39207.25) |
| *a*ˆ1 | 123848.1 | 1679543.9\*\*\* | 93728.0\*\*\* |
|  | (82026.67) | (64722.73) | (3349.72) |
| *a*ˆ2 | 114834.1\*\*\*(1767.47) | 21933.6\*\*\*(1206.64) | 2089.7\*\*\*(60.13) |
| R-squared | 0.9994 | 0.9985 | 0.9992 |
| Observations | 44 | 51 | 51 |

Standard errors in parentheses. Significance Codes,’\*\*\*’ 1%, ’\*\*’ 5%, ’\*’ 10%.

Table 10: Africa Demand Function Estimation

|  |  |  |  |
| --- | --- | --- | --- |
| **Variable** | **Gas** | **Oil** | **Coal** |
| *θ*ˆ1 | -358724340.6\*\*(1.33e+08) | -102194967.8\*\*\*(2.72e+07) | -1078961.6\*\*(3.12e+05) |
| *θ*ˆ2 | 1.61e-4\*\*\*(2.31e-5) | 9.59e-5\*\*\*(1.011e-4) | 3.84e-6\*\*\*(3.88e-07) |

Standard errors in parentheses. Significance Codes,’\*\*\*’ 1%, ’\*\*’ 5%, ’\*’ 10%.

|  |  |  |  |
| --- | --- | --- | --- |
| *θ*ˆ0 | -1.55069e+09\*\*\*(2.97e+08) | 2.64842e+09\*\*\*(2.82e+08) | 81533505.3\*\*\*(1.15e+07) |
| R-squared | 0.96 | 0.58 | 0.82 |
| Observations | 44 | 51 | 53 |

Table 11: Gas Demand Function Estimation

|  |
| --- |
| **Parameter Algeria Egypt Libya Mozambique Nigeria** |
| *θ*ˆ0 1.87e+09\*\*\*(3.54e+08) | -6.82e+08(4.49e+08) | 8.29e+07(7.01e+07) | -2.69e+08(1.84e+08) | -7.42e+08\*\*(2.86e+08) |
| *θ*ˆ1 -1.31e+07 | -4.73e+06 | -2.49e+07\*\* | -5.19e+04 | -1.68e+05 |
| (3.74e+07) | (5.93e+06) | (9.51e+06) | (1.32e+07) | (4.15e+05) |
| *θ*ˆ2 2.00e-5\*\*\* | 3.58e-5\*\*\* | 4.91e-6\*\*\* | 5.91e-6\*\*\* | 2.81e-5\*\*\* |
| (2.92e-06) | (6.14e-06) | (1.19e-06) | (1.51e-06) | (4.14e-06) |
| *θ*ˆ3 |  | -4.28e-2\*\*\*(5.43e-3) |  |  |
| R-squared | 0.70 | 0.87 | 0.94 | 0.90 | 0.92 |
| Observations | 29 | 44 | 20 | 26 | 44 |

Standard errors in parentheses. Significance codes: \*\*\* 1%, \*\* 5%, \* 10%.

*θ*ˆ0: Constant term; *θ*ˆ1: Domestic gas price; *θ*ˆ2: GDP per capita; *θ*ˆ3: Gas net imports.

Table 12: Oil Demand Function Estimation

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Parameter Algeria** | **Angola** | **Egypt** | **Libya** | **Nigeria** |
| *θ*ˆ0 3.80e+08\*\*\* | -1.27e+09\*\* | -6.42e+08 | -1.19e+07 | 7.32e+07 |
| (2.58e+07) | (5.22e+08) | (5.61e+08) | (1.21e+08) | (6.41e+07) |
| *θ*ˆ1 -1.78e+06 | -5.40e+06\*\* | -4.53e+06 | -6.02e+05 | -5.74e+04 |
| (1.04e+06) | (2.02e+06) | (3.01e+06) | (1.86e+06) | (3.68e+04) |
| *θ*ˆ2 6.66e-7(4.60e-7) | 1.07e-5\*\*(4.44e-6) | 2.04e-6(1.25e-6) |  |  |
| *θ*ˆ3 |  |  | 4.10e+04(2.82e+04) |  |
| *θ*ˆ4 | 1.93e+05(1.40e+05) |  |  |  |
| *θ*ˆ5 -7.87\*\*\*(1.54) |  | -7.57\*\*\*(1.32) | -6.36\*\*\*(0.56) |  |
| *θ*ˆ6 | 1.46e+06\*\*\*(3.82e+05) | 7.67e+05\*(4.37e+05) |  |  |
| R-squared | 0.91 | 0.69 | 0.53 | 0.92 | 0.96 |
| Observations | 24 | 27 | 37 | 30 | 39 |

Standard errors in parentheses. Significance codes: \*\*\* 1%, \*\* 5%, \* 10%.

*θ*ˆ0: Constant term; *θ*ˆ1: Domestic oil price; *θ*ˆ2: GDP per capita; *θ*ˆ3: Price of coal; *θ*ˆ4: Rigs world; *θ*ˆ5: Oil net imports; *θ*ˆ6: Electricity consumption from oil.

Table 13: Coal Demand Function Estimation

|  |
| --- |
| **Parameter Botswana Mozambique Nigeria South Zimbabwe****Africa** |
| *θ*ˆ0 1.11e+06(1.34e+06) | -1.11e+07\*\*(5.22e+06) | -5.01e+04(4.00e+04) | 2.26e+08\*\*\*(2.41e+07) | -5.50e+06(1.66e+07) |
| *θ*ˆ1 -2.35e+03 | -1.03e+05 | -2.89e+02 | -1.28e+05\*\*\* | -2.22e+04 |
| (1.92e+03) | (9.10e+04) | (2.63e+02) | (2.90e+04) | (5.35e+04) |
| *θ*ˆ2 3.04e-9(2.00e-8) | 3.35e-7\*\*\*(4.00e-8) | 6.48e-10(0.00) |  | 4.12e-8(1.80e-8) |
| *θ*ˆ3 7.05e+04(5.65e+04) |  |  | 2.94e+06(2.93e+06) |  |
| *θ*ˆ4 |  | 1.87e+02(1.10e+02) |  |  |
| *θ*ˆ5 |  | -2.19e+03(2.83e+03) |  |  |
| *θ*ˆ6 |  |  |  | 1.11e+04\*(5.26e+03) |
| R-squared | 0.80 | 0.81 | 0.76 | 0.46 | 0.40 |
| Observations | 43 | 26 | 23 | 53 | 11 |

Standard errors in parentheses. Significance codes: \*\*\* 1%, \*\* 5%, \* 10%.

*θ*ˆ0: Constant term; *θ*ˆ1: Domestic coal price; *θ*ˆ2: GDP per capita; *θ*ˆ3: Gas price; *θ*ˆ4: Carbon intensity of electricity;

*θ*ˆ5: Greenhouse gas emissions; *θ*ˆ6: Per capita electricity.

### Appendix D:

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1. See Gaudet (2007) for a comprehensive analysis of Hotelling’s rule across various resources, focusing on the market price trajectory of these resources. [↑](#footnote-ref-1)
2. For a given variable $x\left(t\right), \dot{x}\left(t\right)=\frac{dx(t)}{dt}.$ Also $C\_{q}=\frac{∂C}{∂q}$ and $C\_{Q}=\frac{∂C}{∂Q}$ [↑](#footnote-ref-2)
3. The cumulative production $\left(X\_{t}\right) $is obtained by the following formula $X\_{t}=\sum\_{i=1}^{t}q\_{i}$ where $q\_{i}$ represents the annual production in year $i$. The real value $z\_{t}^{r}$ is calculated as $z\_{t}^{r}=z\_{t}^{n}\left(IPC\_{t}/IPC\_{b}\right)$. Where $z\_{t}^{n}$ is the nominal value at period $t$ and base year $b$, respectively. The base year is $2010$. [↑](#footnote-ref-3)
4. The boundary conditions $\dot{Q}\left(T\right)=0$ and $Q\left(T\right)=S\_{0}$ indicate the complete exhaustion of the resource. This conclusion follows from the functional form of the cost function estimated when there is a stock effect. In the absence of a stock effect, complete exhaustion always occurs. [↑](#footnote-ref-4)
5. For the natural gas demand of Algeria, Egypt, and Libya, *r*(*t*) corresponds to the interest rate spread. [↑](#footnote-ref-5)